PROBABILISTIC IMAGE MODELS

Quantitatively describe the uncertain relation between observables and labels in an accepted probabilistic framework

Variables of the Model

Index set $\mathcal{N}$ of spatial coordinates $s = (i, j)$

Unobservable labels $\mathbf{x} = [x_s]_{s\in\mathcal{N}}$ & observables $\mathbf{y} = [y_s]_{s\in\mathcal{N}}$
  x_s: small integer $1 \ldots K$ (i.e. ACR/Fac/QS)
  $\vec{y}_s$: real vector (i.e., the pair (magnetic field, light intensity))

Statistical model given by two distributions $P(\mathbf{x})$ and $P(\mathbf{y} \mid \mathbf{x})$

1: Link to Observables

Make the link via scientist-labeled images and distribution-fitting

For illustration, suppose a (bivariate) Gaussian law is correct:

$$P(\vec{y}_s \mid x_s = k) \sim \mathrm{Normal}(\vec{\mu}_k,\, \sigma_k^2 I)$$

For the QS class ($k = 1$), this in fact fits the SoHO/MDI data well.

2: Quantifying Spatial Smoothness

Typically $\beta > 0$ controls smoothness in

$$P(\mathbf{x}) = \frac{1}{Z} \exp\Big(-\beta \sum_{s\sim s'} 1(x_s \neq x_{s'})\Big)$$

where $s \sim s'$ means: site s close to site s', e.g. one pixel away

Penalty of β per disagreement of nearby pixels

- Objective, automatic inference possible given $\vec{\mu}_k$, σ_k^2, β

INFERRING THE LABELING

Desire *maximum a posteriori* (MAP) estimate

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} P(\mathbf{x}|\mathbf{y})$$

Bayes rule shows $P(\mathbf{x}|\mathbf{y}) \propto P(\mathbf{y}|\mathbf{x})P(\mathbf{x})$
first factor is the traditional "likelihood function"
second is the prior enforcing spatial coherence

Algebra reveals the objective function above is

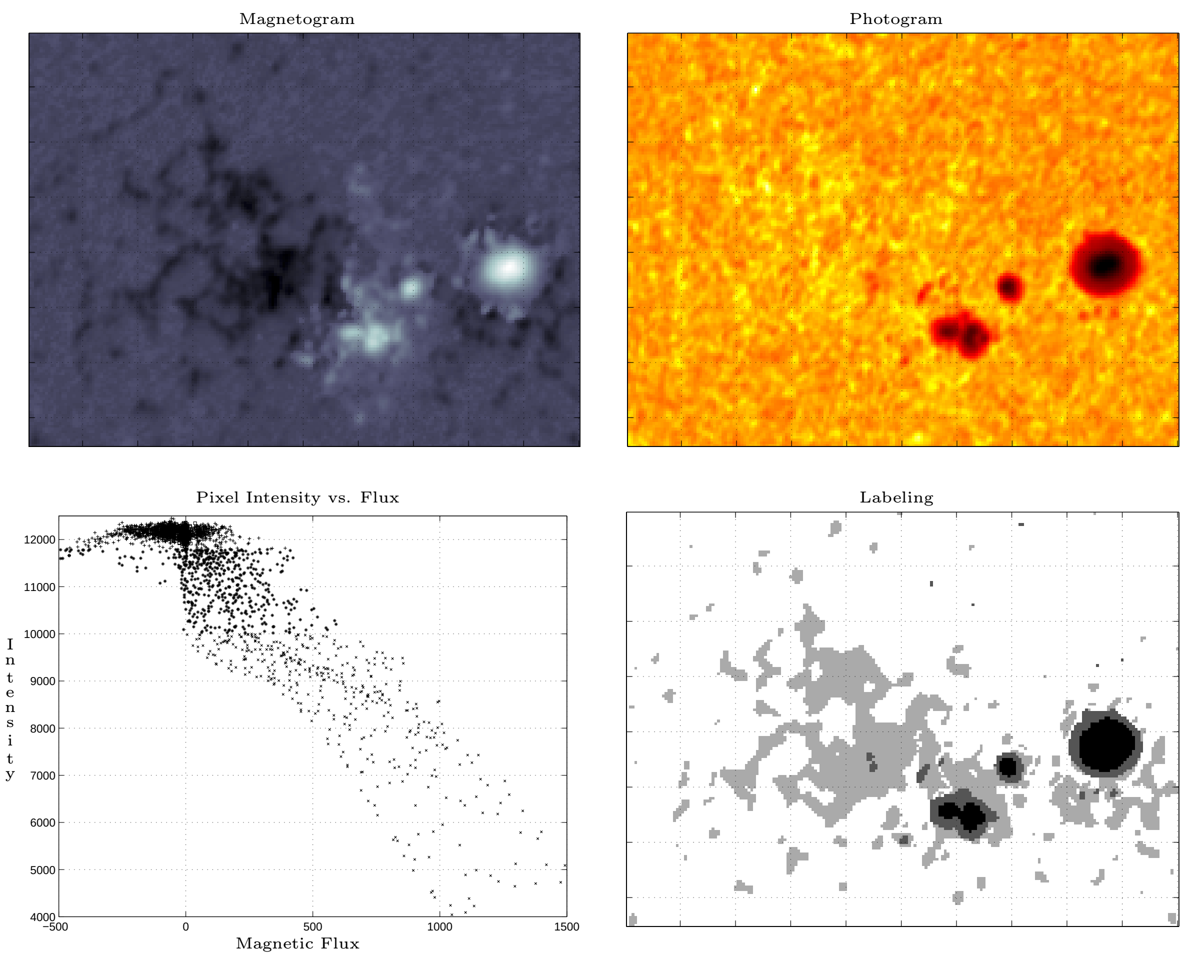
$$\log P(\mathbf{x}|\mathbf{y}) = -\frac{1}{2\sigma^2}\sum_{s\in\mathcal{N}}\left\|\vec{y}_s - \vec{\mu}_{x_s}\right\|^2 - \beta\sum_{s\sim s'} 1(x_s \neq x_{s'})$$

Interpretation
First term: fidelity to data (observation close to its mean)
Second term: image smoothness (this couples the pixel labels)

Maximizing $P(\mathbf{x}\,|\,\mathbf{y})$

- Use 'Gibbs sampler' to *draw* from the distribution $P(\mathbf{x}\,|\,\mathbf{y})$
  Adaptation of stat-mech methods (c.f. Metropolis *et al.* 1953)
  for simulating the state of interacting systems
  Iterative algorithm: starts at some labeling and
  refines it pixel-by-pixel over many iterations
- To *maximize* $P(\mathbf{x}\,|\,\mathbf{y})$, nest G.S. within
  simulated annealing, again parallel to stat-mech ideas
  If 'temperature' $\rightarrow 0$ slowly enough, mode of P is reached
- Considerable problem symmetry makes computation time
  roughly 5 min/image (size 1024^2) on Sun Ultra2 (300MHz).

SAMPLE LABELINGS



- Top, near-simultaneous magnetogram and photogram Taken by SoHO/MDI in August 1996
- Bottom, corresponding scatter plot and labeling
- These were computed using a model which has been fitted to the SoHO/MDI data

SOFTWARE

StarTool

Accessible MRF-based segmentation methods harnessed to SAOtng GUI from the Smithsonian Astrophysical Observatory

Designed for direct use by solar physicists

Uses portable XML (eXtensible Markup Language) files to define the statistical model which is used to label the images

Allows fluid inspection of images and labelings, or scripted batch-mode operation for un-attended labeling

Reads/writes standard FITS format

Extensible by any stand-alone program reading FITS

Labelings and class information (areas, intensities) can be saved for analysis